Problem 5

Verify that, for $t > 0$, $y(t) = \ln t$ is a solution to the differential equation

$$2\left(\frac{dy}{dt}\right)^3 = \frac{d^3y}{dt^3}.$$

Solution

Take derivatives of the given function.

$$\begin{aligned}
y(t) &= \ln t \\
\frac{dy}{dt} &= \frac{d}{dt}(\ln t) = \frac{1}{t} \\
\frac{d^2y}{dt^2} &= \frac{d}{dt}\left(\frac{dy}{dt}\right) = \frac{d}{dt}\left(\frac{1}{t}\right) = -\frac{1}{t^2} \\
\frac{d^3y}{dt^3} &= \frac{d}{dt}\left(\frac{d^2y}{dt^2}\right) = \frac{d}{dt}\left(-\frac{1}{t^2}\right) = \frac{2}{t^3}
\end{aligned}$$

Now plug these formulas into the ODE and check to see if the left side is equal to the right side.

$$\begin{aligned}
2\left(\frac{dy}{dt}\right)^3 &\stackrel{?}{=} \frac{d^3y}{dt^3} \\
2\left(\frac{1}{t}\right)^3 &\stackrel{?}{=} \frac{2}{t^3} \\
2\left(\frac{1}{t^3}\right) &\stackrel{?}{=} \frac{2}{t^3} \\
\frac{2}{t^3} &= \frac{2}{t^3}
\end{aligned}$$

Since this is a true statement, $y = \ln t$ is a solution to the ODE.